Problem 2.1

When a baseball flies through the air, the ratio $f_{\text{quad}}/f_{\text{lin}}$ of the quadratic to the linear drag force is given by (2.7). Given that a baseball has diameter 7 cm, find the approximate speed v at which the two drag forces are equally important. For what approximate range of speeds is it safe to treat the drag force as purely quadratic? Under normal conditions is it a good approximation to ignore the linear term? Answer the same questions for a beach ball of diamter 70 cm.

Solution

The force due to air resistance on a projectile is modelled by

$$\begin{aligned}\mathbf{f} &= -f(v)\hat{\mathbf{v}} \\ &= -(f_{\text{lin}} + f_{\text{quad}})\hat{\mathbf{v}},\end{aligned}$$

where

$$f_{\text{lin}} = bv \quad \text{and} \quad f_{\text{quad}} = cv^2.$$

For spherical objects with diameter D,

$$b = \beta D \quad \text{and} \quad c = \gamma D^2,$$

and in air at STP,

$$\beta \approx 1.6 \times 10^{-4}\ \text{N}\cdot\text{s/m}^2 \quad \text{and} \quad \gamma \approx 0.25\ \text{N}\cdot\text{s}^2/\text{m}^4.$$

Consequently, the ratio of the quadratic drag force to the linear one is

$$\frac{f_{\text{quad}}}{f_{\text{lin}}} = \frac{cv^2}{bv} = \frac{cv}{b} = \frac{\gamma D^2}{\beta D}v = \frac{\gamma}{\beta}Dv.$$

For a baseball and a beach ball,

$$\begin{cases} \text{Baseball:} & \dfrac{f_{\text{quad}}}{f_{\text{lin}}} \approx \dfrac{0.25\ \frac{\text{N}\cdot\text{s}^2}{\text{m}^4}}{1.6 \times 10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}\left(7\ \text{cm} \times \dfrac{1\ \text{m}}{100\ \text{cm}}\right)v \\[2em] \text{Beach Ball:} & \dfrac{f_{\text{quad}}}{f_{\text{lin}}} \approx \dfrac{0.25\ \frac{\text{N}\cdot\text{s}^2}{\text{m}^4}}{1.6 \times 10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}\left(70\ \text{cm} \times \dfrac{1\ \text{m}}{100\ \text{cm}}\right)v \end{cases}.$$

In order to find the approximate speed at which the two drag forces are equally important, set this ratio equal to 1 and solve for v.

$$\begin{cases} \text{Baseball:} & 1 = \dfrac{0.25\ \frac{\text{N}\cdot\text{s}^2}{\text{m}^4}}{1.6 \times 10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}\left(7\ \text{cm} \times \dfrac{1\ \text{m}}{100\ \text{cm}}\right)v \quad \rightarrow \quad v \approx 1\ \dfrac{\text{cm}}{\text{s}} \\[2em] \text{Beach Ball:} & 1 = \dfrac{0.25\ \frac{\text{N}\cdot\text{s}^2}{\text{m}^4}}{1.6 \times 10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}\left(70\ \text{cm} \times \dfrac{1\ \text{m}}{100\ \text{cm}}\right)v \quad \rightarrow \quad v \approx 1\ \dfrac{\text{mm}}{\text{s}} \end{cases}$$

In order to find the approximate speed at which the quadratic drag force is a hundred times greater than the linear one, set the ratio equal to 100 and solve for v.

$$\begin{cases} \text{Baseball:} \quad 100 = \dfrac{0.25 \ \frac{\text{N}\cdot\text{s}^2}{\text{m}^4}}{1.6 \times 10^{-4} \ \frac{\text{N}\cdot\text{s}}{\text{m}^2}} \left(7 \text{ cm} \times \dfrac{1 \text{ m}}{100 \text{ cm}}\right) v \quad \rightarrow \quad v \approx 1 \ \dfrac{\text{m}}{\text{s}} \\ \\ \text{Beach Ball:} \quad 100 = \dfrac{0.25 \ \frac{\text{N}\cdot\text{s}^2}{\text{m}^4}}{1.6 \times 10^{-4} \ \frac{\text{N}\cdot\text{s}}{\text{m}^2}} \left(70 \text{ cm} \times \dfrac{1 \text{ m}}{100 \text{ cm}}\right) v \quad \rightarrow \quad v \approx 10 \ \dfrac{\text{cm}}{\text{s}} \end{cases}$$

Under normal conditions, a thrown baseball and a thrown beach ball travel faster than these speeds. Therefore, the approximate range of speeds that one can ignore the linear drag force is

$$\begin{cases} \text{Baseball:} \quad v \geq 1 \ \dfrac{\text{m}}{\text{s}} \\ \\ \text{Beach Ball:} \quad v \geq 10 \ \dfrac{\text{cm}}{\text{s}} \end{cases}.$$